The Questions (Revised from Hw 1 & 2)

(Deadline 18 Sept at 5 pm)

1. Let $a, b \in \mathbb{R}$ be such that
$$a \leq b + 50\varepsilon \text{ for any } (= \text{all}) \ \varepsilon > 0.$$
Show that $a \leq b$.

2. Let a, b, c be positive real numbers such that
$$a^2 < b < c^2$$
Show that there exists a nature number $N \geq 1997$ and
$$\left(a + \frac{1}{N}\right)^2 < b < \left(c - \frac{1}{N}\right)^2.$$

3. Extend M.I and extended M.I to $\mathbb{Z}$ (regarding that $p(n)$ true for all $n \in \mathbb{Z}$), and show that
$$(m, m+1) \cap \mathbb{Z} = \emptyset \quad \forall\, m \in \mathbb{Z}.$$

4. Let X be a nonempty subset of $\mathbb{R}$ and suppose that
$$\alpha := \inf X \text{ exists in } \mathbb{R}$$
(using inequalities etc write down the definition of $\inf X$). Show that $-\alpha = \sup(-X)$, where $-X := \{-x : x \in X\}$.

5. Let $\emptyset \neq A, B \subseteq \mathbb{R}$ and $A + B := \{a + b : a \in A, b \in B\}$. Show that $\inf(A+B) = \inf A + \inf B$, provided that the inf on LHS exists in $\mathbb{R}$ or the two inf on RHS exists in $\mathbb{R}$.

Pl. submit via Blackboard on or before Friday 18 (at 5 pm)